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CONTRIBUTIONS

of the thesis

INTEGRATION OF DAMAGE DIFFERENTIALS (IDD) FOR FATIGUE LIFE ASSESSMENT UNDER ANY LOADING

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CONTENTS

Since everything in the thesis presented is entirely original and different from all the existing fatigue life investigations, the list of contributions stated below is inevitably long. A trial has been done to reduce the formulations of contributions to less, general paragraphs (in Italic) followed by subparagraphs. Anyway, the text of the contributions below inevitably transforms into an additional summary of the IDD method proposed.

On the occasion of the contributions it is to meanwhile state that every other researcher who exploits already or will exploit the IDD idea (including in some six-dimensional coordinate space of components of variable non-plane state of stress) should recognize the origin of the idea and the author's intellectual property on it.

SCIENTIFIC CONTRIBUTIONS

1. *A new, original scientific research line has been revealed which is radically different from the existing CCA approach in the fatigue life science and which is called Integration of Damage differentials* (*IDD*). *The hitherto existing notion of loading cycle is not basic but particular. In the IDD basis the notion of loading differential lies and damage differential is searched per it. In fact, a completely different concept and strategy for solving the fatigue life problem under any loading is proposed.*

1.1. A new, original idea (conception) has been formulated (in the Preview and further on): to follow the ds differentials (the Δs ~ ds little finite differences) of the loading (stressing) and to compute dD ~ ΔD damage differential per each new ds ~ Δs differential appeared; the fatigue life ends when the sum (the integral) of the damage differentials reaches a critical value.

1.2. It has been revealed that the IDD concept is radically different from the hitherto existing so-called Cycle Counting Approach (CCA) in the fatigue science where the loading variability is only associated with the notion of loading cycle which is close to mind. With IDD, the loading variability has been introduced as appearance of ds ~ Δs differential during dt ~ Δt differential at current (running) time t. This is a completely different way of thinking based on the Calculus. This way corresponds to the achievements of the contemporary sciences to which the differential and integral analysis is applied (on this occasion, the Treatise on the differentials and integrals has been written).

1.3. A valuation has been made (in Subchapter 1.5) of the up-to-date state of fatigue life investigations (prior to involving IDD). A decisive conclusion has been derived that the fatigue life problem does not have any universal and uniform solution. The notion of loading cycle played its progressive role under the simple cyclic loading presented by an only oscillogram $s(t)$ in which a variation between s_{min} and s_{max} repeats periodically. However, upon complicating the loading and trying to continue with the way of thinking based on the limited and particular notion of loading cycle, no clear, logical and uniform fatigue life assessment method is created under any (general) loading. Instead, numerous fatigue life criteria are proposed, and efforts are scattered in a great scale.

1.4. The point has been revealed where the way of thinking based on the notion of loading cycle fails: in a two-dimensional or three-dimensional coordinate space of arbitrary mutual variation of components of the so-called general (any) loading (Fig. 1.1-3b), no reversal or some other indication for a cycle could be considered. The notion of loading cycle is only applicable under a single oscillogram (i.e. under so-called *r*-loading). This is the prime cause of all the trials done in order to reduce two or three or more oscillograms to a single oscillogram in which cycles could be counted. On the background of the numerous investigations for cycle counting in a non-cyclic oscillogram, IDD surprisingly shows all that as being not definitely necessary.

2. *It has been revealed that the great theoretical and applied fatigue life problem is generally and uniformly solvable namely based on the IDD concept in accordance with the contemporary achievements of numerical integration of any differential equations in finite differences or finite elements under any integration conditions, by means of the contemporary computers. Thus, nowadays, it is already possible to redirect the accumulated world CCA experience to IDD.*

2.1. It has been revealed (in Preview and Subchapter 1.5) that, for fatigue life assessment under general (the most complicated) loading, the main advantage of Calculus should be used: universal integration of a differential equation under any (including the most complicated) integration conditions. This means, differentials d*D* can be integrated directly (without searching for cycles, decomposition of loading etc.), universally and uniformly under any oscillograms $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ which plainly take the role of arbitrary integration conditions.

2.2. It has been revealed (in Preview and further on) that the IDD concept could not have been implemented prior to the computer era, in case the oscillograms are more than one, since the integration should be done numerically in little finite differences. Correspondingly, the IDD idea has come as a result from the contemporary computer way of thinking. Otherwise, the same idea could have been realized without computers, as well, but under a single non-cyclic oscillogram.

2.3. Acceptability and continuity of the accumulated world CCA experience has been enabled in the thesis. IDD does not reject that experience at all but uses it, as follows. At the IDD input, basic traditional fatigue life characteristics are entered: *S-N* lines under cyclic *r*-

loadings with different values of *k*. In other words, all the former knowledge and criteria, regarding such lines, stay at the IDD input. As well, any other CCA achievements in particular cases of non-proportional loadings can stay at the IDD input (see also contribution 13.4). In fact, IDD throws a bridge for linking all the incompatible models and criteria under specific loadings using them in a different way under any (general) loading.

3. *The first IDD problem has been formulated (Subchapter 1.6) and solved (Subchapter 2.1): under arbitrary and non-proportional oscillograms $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$, how to redefine the variant* d*s differential from the* σ_x-σ_y-τ_{xy} *coordinate space (Fig. 1.1-3b) into a differential* d*s which is invariant of the axes x and y. The idea of the transforming ellipse has been generated. The invariant* d*s differential proves to be in a new, special three-dimensional space* σ'-σ''-dτ *where* σ' *and* σ'' *are the principal stresses, and the third dimension* dτ *is infinitesimal. In that three-dimensional space, an invariant trajectory is described by adding each next invariant* d*s differential. The latter has two components in the* σ'-σ'' *plane, and the third* dτ *component is perpendicular to the* σ'-σ'' *plane. That third component is the most interesting: it takes into account the rotation of the principal axes. The invariant* d*s differential revealed results from single possible logic. It leads to new terminology of the loading kinds.*

3.1. Original differential geometrical analysis has been done which reveals the transforming ellipse and leads to formulating d*s* in the new σ'-σ''-dτ coordinate space. An essential point in the analysis is the idea of taking into account the rotation of the principal axes which reveals the third dτ component of d*s* that is perpendicular to the σ'-σ'' plane. And, in the σ'-σ'' plane, the components ds_r and ds_c of d*s* have been introduced: ds_r is called radial differential and ds_c is called circumferential differential. Concomitantly, the physical sense of the differentials d*s*, ds_r, ds_c and dτ has been revealed.

3.2. Hence, new term appear regarding important, characteristic, basic loading kinds: (pure) *r*-loading (radial loading, proportional or one-component loading) containing ds_r differentials only, (pure) *c*-loading (circumferential loading, non-proportional, without rotation of the principal axes) containing ds_c differentials only, and (pure) dτ-loading containing dτ differentials only where the principal axes rotate but the principal stresses remain constant. Trajectory ratios t_r, t_c and t_τ have been introduced which, under the three pure basic loadings, are (1, 0, 0), (0, 1, 0) and (0, 0, 1). Any mixed loading has $t_r < 1$, $t_c < 1$ and $t_\tau < 1$.

3.3. It has been revealed that, besides the invariant space of stresses σ'-σ''-dτ, IDD can also be developed in an invariant space of strains ε'-ε''-dγ, as well as in two-dimensional spaces under two oscillograms. The thesis has been limited to the σ'-σ''-dτ space of plane stressing but the possibility to exploit the IDD idea in the other spaces mentioned, or analogously in six-dimensional spaces of non-plane stressing, is also in the scope of the intellectual property stated.

4. *Fatigue damage differential* dD_r = R_rds_r *has been introduced* (*Subchapter 2.2*) *per the* ds_r *differential and thus a new basic notion has been revealed: damage intensity* $R_r(s)$ = d$D_r(s)$/ds_r *under r-loading as a derivative of damage function* $D_r(s)$. *The argument s is the distance from* ds_r *to the coordinate origin of the* σ'-σ'' *plane, at a given value of* $k = \sigma''/\sigma'$. *The problem of determination of* $R_r(s)$ *arises. It has been solved* (*in Subchapter 2.3*) *based on a so-called input* R_r *prototype in a way allowing IDD to reproduce a given S-N line under cyclic r-loadings having one and the same k value. Further on, the problem of determination of* R_r *in the whole* σ'-σ'' *plane arises. It has been solved* (*in Subchapter 2.4*) *in a way allowing IDD to reproduce two or more given S-N lines based on corresponding* R_r *prototypes under cyclic r-loadings having different values of k. With that, the notion of fatigue limit has been replaced (in Section 2.7.7*) *by the more general notion of* L_r *limiting line which surrounds* L_r *area of* $R_r = 0$ *in the* σ'-σ'' *plane* (*L_r is no-damage area where the* ds_r *differentials do not cause damage*).

4.1. It has been revealed that the Newton-Leibniz theorem gives a remarkable opportunity for determination of $R_r(s)$ intensity: the equation (2.3.2-1) has been deduced.

4.2. It has been revealed that the traditional break of an *S-N* line in two at the fatigue limit means implication of a big impulse (infinitely big, theoretically) in the R_r intensity what represents the break as incorrect idealization. Nevertheless, the possibility has been developed for IDD to reproduce a given *S-N* line as a breaking one in so-called breaking mode or impulse mode. Then, the R_r prototype coincides with the given *S-N* line. Otherwise, the so-called smooth (smoothing) mode has been postulated as basic and more logical mode, and as a new notion: IDD reproduces a given smoothly bending *S*-*N* line. Then, the R_r prototype does not coincide exactly with the given *S*-*N* line but that line smoothly bends above it. The equation (2.3.4-5) of the smooth *S*-*N* line has been deduced.

4.3. A solution of the problem of R_r determination in the whole σ'-σ'' plane has been achieved by approximation of lines of equal lives under cyclic *r*-loadings at different *k* values involving elliptic arcs. Heavy equations have been deduced for R_r determination based on successive approximations: the equation (2.4.7-5) and then the equation (4.1.9-7) preceded by several other equations.

4.4. The new area of $R_r = 0$ has been defined as surrounded by an L_r line of equal life of N_r cycles from the R_r prototypes in smooth mode, or surrounded by an L_l line of equal life of N_l cycles in impulse mode. Correspondingly, new terms are introduced: L_r no-damage area surrounded by limiting L_r line, and L_l no-damage area surrounded by limiting L_l line. Instead of fatigue limits, the N_r number in smooth mode or the N_l number in breaking mode is entered at every *k* value. The through-points and the slopes of the input R_r prototypes are also entered.

4.5. All the mathematical problems have been solved (in Subchapter 4.1 in continuation from Subchapter 2.4) enabling algorithmic procedures for computation of the damage differential $\mathrm{d}D_r = R_r\mathrm{d}s_r$ in case $\mathrm{d}s_r$ is out of the L_r area (or the L_l area). In the impulse mode (not recommendable but admissible), a sudden addend is added to the cumulative damage in case the trajectory crosses the L_l line.

5. *Fatigue damage differential* $\mathrm{d}D_c = R_c\mathrm{d}s_c$ *per* $\mathrm{d}s_c$ *differential has been introduced and thus a second kind of damage intensity* $R_c = \mathrm{d}D_c/\mathrm{d}s_c$ *under c-loading has been revealed. An idea has been developed* (*in Subchapter 2.5*) *for determination of* R_c *in the entire* σ'-σ'' *plane what has involved aspects unrevealed before in the CCA investigations. Therefore, at the present stage of IDD developments, in following the logic of building the IDD theory,*

conditional R_c prototypes have been introduced. They are analogous to the R_r prototypes and thus the R_c intensity is set in comparison to R_r. The ratio $f_c = R_c/R_r$, averaged in the σ'-σ'' plane, has been introduced. It takes the role of empirical factor of sensitivity of the material to non-proportional c-loadings in comparison to r-loadings.

5.1. A differential equation (2.5.1-1) for R_c has been deduced involving questions which do not meet any solutions in the existing world research experience. Respectively, ideas for new kinds of investigations have been generated. They would put the existing fatigue life criteria under a new situation under *c*-loading.

5.2. The R_c intensity has been set in the same way like R_r: by means of (conditional) R_c prototypes. An expectation has been grounded that $R_c \geq R_r$ and, correspondingly, $f_c \geq 1$. With that, f_c could be significantly greater than 1. How great this factor would experimentally prove under immovable principal axes is a new question causing interest.

6. *Fatigue damage differential* $\mathrm{d}D_\tau = R_\tau \mathrm{d}\tau$ *per* $\mathrm{d}\tau$ *has been introduced. Thus, a third kind of damage intensity* $R_\tau = \mathrm{d}D_\tau/\mathrm{d}\tau$ *has been revealed under* $\mathrm{d}\tau$*-loading i.e. under constant principal stresses in rotating principal directions* (*principal axes*). *This loading* (*Subchapter 2.6*) *came as a discovery thanks to the IDD point of view. It will put to the test all the existing fatigue life criteria. That is why it is appealed to laboratories to enable research under* $\mathrm{d}\tau$*-loading provided that they have the necessary experimental facilities. An idea for determination of* R_τ *in the entire* σ'-σ'' *plane has been developed. Due to the lack of previous investigations, conditional* R_τ *prototypes have* (*again*) *been introduced analogously to the* R_r *prototypes. Thus, the* R_τ *intensity is set* (*again*) *in comparison to* R_r. *The ratio* $f_\tau = R_\tau/R_r$, *averaged in the* σ'-σ'' *plane, has been introduced. It takes the role of empirical factor of sensitivity of the material to rotation of the principal axes.*

6.1. The true sense of the experiment of Findley et al. with rotating disc, done a half a century ago, has been revealed (in Section 2.6.1) as the only experiment under (pure) $\mathrm{d}\tau$ loading although with limited options.

6.2. Another technical implementation of $\mathrm{d}\tau$ loading has been proposed (in Section 2.6.2) based on the revealed elliptical variant trajectory of the $\mathrm{d}\tau$ loading.

6.3. An equation (2.6.3-3) for the R_τ intensity has been deduced in case experimental life under $\mathrm{d}\tau$ loading is known.

6.4. The dτ loading has been revealed (in sections 2.6.5 and 2.6.6) as a maximized case in comparison to other cases of 'weaker' loadings. The comparison is not in favor of the concept of one critical plane and puts to the test all the existing fatigue life criteria.

6.5. Together with introducing (conditional) R_τ prototypes in comparison to the R_r prototypes, an expectation has been grounded that $R_\tau \geq R_r$ and, correspondingly, $f_\tau \geq 1$. With that, f_c could be significantly greater than 1. How great this factor would experimentally prove under the rotating principal axes of (pure) dτ-натоварване is the next new question causing interest.

7. *A basic version of formula of* dD ~ ΔD *has been proposed* (*in Subchapter 2.7*) *and grounded under any mixed loading where* ds_r, ds_c *and* dτ *simultaneously appear. The formula is:* $R_r(\mathrm{d}s_r^2 + f_c^2\mathrm{d}s_c^2 + f_\tau^2\mathrm{d}\tau^2)^{1/2} \sim R_r(\Delta s_r^2 + f_c^2\Delta s_c^2 + f_\tau^2\Delta\tau^2)^{1/2}$. *This formulation gives the attractive opportunity to use the traditional S-N lines under cyclic r-loadings for fatigue life assessment also under any non-proportional loading thanks to the factors* f_c *and* f_τ *of the material sensitivity to loading non-proportionality. The task has been put for building, enlarging and precising an empirical data bank for* f_c *and* f_τ *as a result of IDD verifications. Within that bank, the values of* f_c *and* f_τ *are to be appropriately selected to serve for every new application of the IDD method developed. Together with* f_c *and* f_τ, *the parameters* N_c *and* N_τ *in ratios to* N_r *will also participate in the data bank. They set the areas* L_c *and* L_τ *in the* σ'-σ'' *plane where the differentials* ds_c *and* dτ *cause no damage. Those areas are surrounded by limiting lines* L_c *and* L_τ *replacing the previous notions of fatigue limits under non-proportional loadings.*

7.1. The main advantage of the formula of 'mixed' differential $\mathrm{d}D = R_r(\mathrm{d}s_r^2 + f_c^2\mathrm{d}s_c^2 + f_\tau^2\mathrm{d}\tau^2)^{1/2}$ is: it reduces to $\mathrm{d}D_r = R_r\mathrm{d}s_r$, $\mathrm{d}D_c = R_c\mathrm{d}s_c = f_cR_r\mathrm{d}s_c$ and $\mathrm{d}D_\tau = R_\tau\mathrm{d}\tau = f_\tau R_r\mathrm{d}\tau$ under pure r-loading, c-loading and dτ-loading. The mixed differential proposed leads to concrete author's IDD life equation (2.7.5-1). This means, other authors can also propose and study future different formulae of dD, and revealing a better version is not excluded. In this sense, the contributions counted above are general for the IDD concept, and now concrete contributions of the concrete IDD method proposed are stated. Below, the next concrete contributions are easy to distinguish while the rest contributions are general for the IDD concept. The author also reserves the right over the other IDD concretizations based on the equations (2.7.2.1), (2.7.3.3), (2.7.4.1) and (2.7.4.2) (the latter two equations relate also to strain coordinates).

7.2. Considering the too complicated and inscrutable mechanism of fatigue under any (general) loading, it is not realistic to assume that the factors f_c and f_τ under the (pure) c-loading and dτ-loading would remain the same in any mixed dD differential. Therefore, a concrete contribution is the empirical data bank envisaged separately for f_c and f_τ of the first practical category of non-proportional loading, with future inclusion of f_c separately for the second category and of the 'pure' f_c and f_τ separately under the pure c-loading and dτ-loading. The 'pure' f_c and f_τ will have ultimate values in intervals where the 'mixed' f_c and f_τ will vary. The 'pure' f_c and f_τ will be obtained in the future if researchers in proper laboratories adopt the IDD point of view and start enabling experiments under pure c-loading and dτ-loading. The thesis limits its concrete contributions to the first practical category of non-proportional loadings. Separately, initial f_c data of the second practical category can be derived from the publication [171]. And, in Volume II, the work continues for enlarging and précising the empirical data bank of the IDD parameters.

7.3. The ideas of the L_c and L_τ areas and lines are general contributions. These areas and lines could be formed by other authors in different, more complicated but hardly exacter manners. In the thesis, the L_c and L_τ lines are simply (conditional) lines of equal lives N_c and N_τ to which the R_c prototypes and the R_τ prototypes are extrapolated. Thus, an only given parameter N_c serves for forming the whole limiting L_c line that surrounds the L_c area of $R_c = 0$, and an only given parameter N_τ serves for forming the whole limiting L_τ line that surrounds the L_τ area of $R_\tau = 0$.

8. *It has also been revealed* (*in Section 2.4.2 etc.*) *that the IDD concept allows a simple and uniform approach to taking into account non-zero static* (*mean*) *stresses* $\sigma_{x,\mathrm{m}}$, $\sigma_{y,\mathrm{m}}$ *and* $\tau_{xy,\mathrm{m}}$ *of the oscillograms* $\sigma_x(t)$, $\sigma_y(t)$ *and* $\tau_{xy}(t)$: *based on* $\sigma_{x,\mathrm{m}}$, $\sigma_{y,\mathrm{m}}$ *and* $\tau_{xy,\mathrm{m}}$, *an equivalent*

static stress $\sigma_{\text{equ,m}}$ *is calculated and the input prototypes of the damage intensity are adapted to* (*are made compatible with*) *that* $\sigma_{\text{equ,m}}$. *In comparison to* $\sigma_{\text{equ,m}} = 0$, *a non-zero* $\sigma_{\text{equ,m}}$ *moves up the* R_r*-prototypes* σ'_{max}-N (*and increases their indicators of slope*) *according to the well-known Smith relations, and thus the* R_r *intensity decreases. The accumulated world experience about S-N lines valid for non-zero levels of cyclic r-loadings is applicable again. But now it is applicable directly, without any additional conceptual complications and contradictions that appear with CCA upon considering the so-called means stress effect.*

8.1. Adaptation (compatibility) of the input prototypes to a line of equal $\sigma_{\text{equ,m}}$ in the σ'-σ'' plane has been worked out. The choice of a criterion for that line is left to the IDD user's competence. The classical criteria of Von Mises, Tresca and Mohr can be used, as well as any other empirically revised criterion. Besides, the IDD manner of composing the lines of equal lives as consisting of elliptic arcs suggests an additional proposal for that line of equal $\sigma_{\text{equ,m}}$: to be composed of elliptic arcs again. The equations (2.4.2-5) – (2.4.2-8) serve for that, with participation of available material static strength measures at different values of k.

8.2. In case no experimental *S-N* lines are available in compatibility with $\sigma_{\text{equ,m}}$ to serve for input R_r-prototypes, then the IDD user can create hypothetical *S-N* lines based on competence in Smith relations. Besides, the IDD user could prefer the proposed equation (1.2.2-1) for generating a hypothetical *S-N* line based on a middle solution between the Gerber parabola and Goodman line.

8.3. With IDD, once $\sigma_{\text{equ,m}} \neq 0$ is taken into account at the input prototypes, then its influence on the dD differentials runs automatically. Whereas, with CCA, the different trials to reduce the three oscillograms $\sigma_x(t)$, $\sigma_y(t)$, $\tau_{xy}(t)$ to a single oscillogram and to search for cycles in it, and the different opinions on how to involve the non-zero $\sigma_{x,\text{m}}$, $\sigma_{y,\text{m}}$ and $\tau_{xy,\text{m}}$, lead to additional conceptual complications and contradictions. That is why the CCA studies are mostly limited to cases of zero $\sigma_{x,\text{m}}$, $\sigma_{y,\text{m}}$ and $\tau_{xy,\text{m}}$. The review of the thesis has shown that the problem of non-zero $\sigma_{x,\text{m}}$, $\sigma_{y,\text{m}}$ and $\tau_{xy,\text{m}}$ remains unclear even in the simple case of rotating bending combined with constant torsion (Section 1.4.6). And, IDD has not had any problems in that case and has been successfully verified (in Subchapter 5.8). In the work continuing in Volume II, IDD has given again satisfying computed lives in a case of $\sigma_{\text{екв,m}} \neq 0$, whereas another CCA method has not.

8.4. When analyzing CCA (under a single oscillogram) from the IDD point of view based on the notion of damage intensity, an objection appears to the many proposed manners of transforming the different $s_{a,i}$ amplitudes according to the different $s_{m,i}$ mean stresses. The objection is: from one range (double amplitude) to the next one, those manners cause discontinuities (jumps) of the damage intensity function which is, however, physically continuous.

9. *The IDD way of thinking contributes* (*in sections 1.3.4, 2.3.7, 2.7.6 etc.*) *to perception of the real fact that only the loading history to the current t time influences* d*D damage differential during* d*t, and the loading future after the t time, what "the material does not know", cannot influence* d*D. In this regard, CCA actually admits illogicality called in the thesis "paradox of peeping into the loading future". The IDD viewpoint turns thinking in a more logical direction for making out the inscrutable mechanism of influence of static stresses being formed, for taking into account the non-linearity of damage accumulation, for the true damage intensity, etc. With that, new interesting questions, unconsidered before, appear.*

9.1. The IDD way of thinking helps for realizing that the damage accumulation process should be considered such as it is: settled to the t time, continuous, differential by differential, considering the fact that "the material does not know" the loading future e.g. a future reversal (in a single oscillogram). It is the CCA researcher who "peeps" in the loading future to see the reversal as the end of a range, then to be back to the beginning of the range and form $s_{a,i}$ amplitude, discontinuing the damage intensity function in this way, avoiding the question what the damage is along a part of the range, and so on. More generally speaking, the researcher preliminarily "peeps" into an entire non-cyclic oscillogram and builds cycles between reversals that happen at different times (being also far from each other), and only then does the researcher evaluate the accumulated damage. But it meanwhile accumulates obviously without following such procedures of the researcher.

9.2. The concrete IDD method proposed also admits the "paradox of peeping into the loading future" in case $\sigma_{x,m}$, $\sigma_{y,m}$ and $\tau_{xy,m}$ are wanted in advance. However, the direction of thinking is more logical. Actually, even at this stage, the IDD user can proceed only according to the loading history to the current t time. This means, at $t = 0$ the user can set that damage intensity which is valid for zero static stresses and then track non-zero static stresses $\sigma_{x,m}(t)$, $\sigma_{y,m}(t)$ and $\tau_{xy,m}(t)$ being formed to t. And here the interesting question arises, not asked before, what is the time interval which precedes t and is sufficient for the material to "remember" $\sigma_{equ,m}$ being settled and "get accustomed" to it? In the CCA language under a single $s(t)$ oscillogram, the question is: in how many previous ranges "does the material remember and get accustomed to" s_m being settled? And at the very beginning of the loading, how much is the damage per the first range? Or how much is the damage per a part of the first range?

9.3. A general view on all the possible arguments of the damage intensity function is presented (in Section 2.7.6) associating those arguments with the current t time and with the loading history to that t only. It is shown that the very current cumulative damage $D_\Sigma(t)$ can be an argument of the damage intensity and can recurrently cause non-linearity of the damage accumulation (if such non-linearity is to be taken into account).

10. *The IDD concept has been also developed* (*in Subchapter 2.8*) *in statistical* (*probabilistic*) *interpretation under any random loading. The idea of the* d*s differentials and their components* ds_r, ds_c *and* dτ *leads simply and logically to a possibility for their direct statistical distribution into one or more nets* (*grids, meshes*) *in a suitable IDD space or an IDD plane. Relations have been deduced between the statistical frequencies or the probabilistic characteristics of the distribution and the damage intensity. Based on them, the fatigue life can be computed. Thus, a new, statistical and probabilistic IDD scientific line of research has separately been opened.*

10.1. IDD allows, without any conceptual problems, statistical and probabilistic interpretation of the mutual run of multiaxial loading components: that interpretation relates to the instantaneous (current) values of the components. CCA cannot conceptually achieve the same since CCA can enable statistical distribution of cycles and amplitudes only but such cannot be defined in the mutual run of the loading components.

10.2. In particular, under random r-loading (having a single oscillogram) where CCA introduces distribution of amplitudes, IDD introduces one-dimensional distribution of the instantaneous values. The relations between those two kinds of distributions have been revealed. Thus, the accumulated existing experience of the probabilistic approach as a part of CCA related to distribution of amplitudes can be transmitted now to distribution of instantaneous values.

11. *For the IDD purposes, and also as a contribution to the theory of variable stresses and strains out of IDD, an original differential theory has been developed suggested by the IDD idea and by the* d*s differential. Within the frames of the theory in question, the following problems have been solved: determination of the* $\alpha'(t)$ *function of rotation of the principal axes and the related determination of the principal stresses and strains functions by correct switchover of the* ± *signs in their equations.*

11.1. Associated with the $\alpha'(t)$ function and the switchover of the ± signs, new notions and original interpretations have been involved. Three conditions of switching-over have been derived and formulated (sections 2.4.4 and 4.1.2 – 4.1.4) for the first time. A necessity of dividing variant elements into two sub-elements has been found out. The very problem of such dividing has also been solved (Section 4.1.5). And, having the solution for the $\alpha'(t)$ function achieved, the way for IDD application to orthotropic materials is also opened.

11.2. It has been proved that the invariant trajectory of the principal stresses (or of the principal strains) passes onto the other side of the η axis only in case the variant trajectory exactly crosses that axis. Even an infinitely little deviation of the variant trajectory from the η axis leads to sending the invariant trajectory back to the same side of that axis (Section 4.1.5).

12. *A reduced IDD fatigue life equation* (3.1.2-1) *has been deduced for finding N in the particular case of non-cyclic loading. The equation allows computation of N without any apparent participation of the* d*s differentials but only based on the Newton-Leibniz theorem. With that, the primitive* $D(s)$ *damage function participates together with the no-damage interval* L_r *(or* L_l*). Thus, under a single non-cyclic oscillogram, the fatigue life computation has been enabled directly, without looking for cycles and their counting, surprisingly to the CCA developed for more than one century.*

12.1. A theoretical analysis has been done (in Subchapter 3.2) revealing what comparative results may be obtained according to IDD and rain-flow CCA, and under which conditions.

12.2. It has been theoretically proved (in subchapters 3.2 and 3.4) that IDD successfully 'takes the examination' under a single oscillogram and is able to assess the fatigue life even better than CCA under the conditions of the different disputable CCA versions.

13. *Eventually, the developed IDD theory allows a new strategy, as follows. After it seems unrealistic that one of the many proposed CCA life criteria would prove to be always all-applicable and all-valid under any concrete non-proportional loadings of the first and second category, as well as under the unstudied c-loadings and* (*especially*) dτ*-loadings revealed in the thesis, and that the same criterion would hardly give* $(N_{\text{cmp}}/N_{\text{exp}})_{\text{average}}$ *sufficiently close to* 1 *in numerous and heterogeneous verifications, then: it seems* (*Subchapter 5.1*) *more realistic and more practical to proceed from* $(N_{\text{cmp}}/N_{\text{exp}})_{\text{average}}$ *sufficiently close to* 1 *and, based on this, to canalize the investigations into building an all-valid empirical data bank of successful IDD parameters. Such a "reversed" strategy is only possible with IDD since the* N_{cmp} *life can always be computed now in a uniform and universal way under any loadings.*

13.1. The thesis gives a new theoretical viewpoint and light in which the immanent failure of each fatigue life criterion is clearer seen in case the notion of cycle of a simple cyclic oscillogram is tried to be validated for three arbitrary and non-proportional oscillograms. According to the theory and the way of thinking in the thesis, loading cases (maximized, minimized etc.) can always be found where the criterion will prove to be either inapplicable or conflicting. There is not any CCA criterion having applicability and validity proved simultaneously under the non-proportional loadings of both the first and second practical category, and under the most general loading with three arbitrary and non-proportional oscillograms, and now under the characteristic pure *c*-loading and (especially) pure dτ-loading revealed by IDD.

13.2. The IDD concept is always applicable under any loading since it is released from the limiting notion of loading cycle. IDD is actually not any life criterion in the sense which is meant for such a criterion. The IDD concept gives a way for direct application of R_r damage intensity from cyclic *r*-loadings to any other non-cyclic *r*-loading, and, thanks to the empirical parameters f_c and f_τ proposed, the R_r damage intensity becomes applicable under any non-proportional loading, as well. It is expected that f_c and f_τ, derived from the condition $(N_{\text{cmp}}/N_{\text{exp}})_{\text{average}} \to 1$ separately for the two categories of non-proportional loadings and for the pure *c*-loading and dτ-loading, could be reliably selected from respective data banks built. These banks will be parts of an all-valid data bank. In these banks, certain laws of variation of f_c and f_τ in certain intervals will undoubtedly emerge, although scattering. Such laws and intervals will emerge, as well, for the proposed parameters N_c and N_τ in ratios to N_r which will also be placed in the data banks together with f_c and f_τ. It has been at least revealed that the no-damage areas L_c and L_τ are closer to the coordinate origin than the L_r area. And even if the data-bank values of the proposed IDD parameters are not reliable enough, they can always be set higher in favor of safety.

13.3. The selection of f_c, f_τ, N_c and N_τ from the intervals of variation in the data banks provides that flexibility and general validity of IDD which the existing criteria miss. The fact that some criteria are confirmed in given concrete cases of non-proportional loadings but are inapplicable in other cases where different criteria are proposed corresponds to the fact that the IDD parameters f_c, f_τ, N_c and N_τ vary.

13.4. Upon building an all-valid data bank of the IDD parameters f_c, f_τ, N_c and N_τ, besides setting empirical values, every life criterion with confirmed partial validity under specific non-proportional loadings can be used. According to fatigue life and/or fatigue limit assessed by such criterion, IDD can form f_c and f_τ and/or N_c and N_τ.

13.5. The "reversed" strategy is, on principle, impossible for any CCA criterion due to its specific application to concrete loading. The criterion cannot deal, under any loading, with selectable general parameters of the kind of the IDD parameters f_c, f_τ, N_c and N_τ.

13.6. A possibility that other researchers could reveal more successful IDD parameters is not excluded. The most important contribution and effect of the thesis will be canalizing the other researchers' investigations into IDD.

THEORETICAL AND APPLIED CONTRIBUTIONS

1. *Mathematical and algorithmic problems of a great number have been solved* (*in Chapter 4*) *for computer implementation and application of the author's IDD method. The problems do not have existing analogues and have been put for the first time.*

1.1. The two systems of interpolation set (in Subchapter 2.9), trigonometric and spline ones, have been mathematically and algorithmically worked out entirely. They have been incorporated in the IDD software and, when necessary, they automatically generate n_i–1 additional ordinates between every two successive original ordinates of the oscillograms. The IDD user can enter a suitable n_i number for interpolation in case the original ordinates are at rare intervals, the more proper interpolation can be chosen in any case considered, and so on.

1.2. The new mathematical instruments for building the Δs elements have been entirely worked out aimed at programming and application. A generalized X-Y coordinate plane has been introduced and the perpendicular components of the Δs elements are added. Thus, the software has been enabled to proceed in both the σ'-σ''-$\Delta\tau$ and ε'-ε''-$\Delta\gamma$ coordinate space, as well as with entering both three and less than three non-zero oscillograms.

1.3. The specific mathematical and algorithmic problems appeared in terms of conditions of ± switchover have been entirely solved, including conditions of providing continuity of the invariant trajectory over the η axis. Simultaneously, the IDD user has been given the alternative option to cause illustrative discontinuity over η what is convenient in certain cases of IDD application. Besides, together with non-connecting the Δs_{xy} elements in the X-Y plane as an option by default, the alternative option of connecting the Δs_{xy} elements has been mathematically also provided. The different options, as well as the choice of the n_i number for interpolation, enable an additional control of the results and more responsible computation in a more precise regime, or, vice versa, draft results can been obtained, etc.

1.4. The alternative impulse mode (breaking mode) has been mathematically and algorithmically worked out for the case when the IDD user would prefer using this mode at crossing an L_l (instead of L_r) limiting line.

1.5. Lots of additional mathematical and algorithmic details and difficulties have been overcome in reference to: investigation of the possibility for the current elliptic equation to be solved (this has been also an individual, uniquely put and solved problem concerning the correctness of the lines of equal lives and hence the correctness of the given *S-N* lines); the very solution of the equation by successive approximations based on two methods, a basic and a reserve ones, specifically applied for IDD; the follow-up computation of the corresponding damage intensity; etc.

1.6. Lots of additional issues have been conceptually solved, and mathematically and algorithmically developed, in reference to: the contents of the additional and control results; the messages given by the software during the dialog with the IDD user; the graph mode organization; etc.

2. *Unique IDD software called Ellipse has been created. For that, algorithmic and programming problems have been solved in such a volume and complexity which, under different conditions, would engage a whole team of mathematicians and programmers.*

2.1. The creation of the software has gone through a lot of problems for the lack of existing analogue, through various versions, perfecting, involving new program modules, tests, overcoming algorithmic traps, and so on. The software creation has been done by a single person since the other colleagues have wanted to see results first and then possibly support IDD. Thus, the algorithmic and programming efforts have taken about 30 years in all. The beginning was prior to 1980 with use of the then algorithmic Fortran 77 language and the continuation has been done in the same language. This activity has been of the kind of creation of the contemporary program packages for solutions of the differential equations of the elasticity theory and other theories in finite elements done by teams of highly qualified mathematicians and programmers.

2.2. The software is confirmed as well-working. Its interface in its present working form enables properly the dialogue between the IDD user and the computer but not in a modern form suitable for licensing. That is why the software is freeware. However, if the IDD method, practically implemented thanks to this working software, makes its way everywhere in the world, then the importance of this IDD-software contribution will sharply increase. A team for creating licensed IDD somewhere will be formed somewhere in the world and marketing will follow up. Any team starting such activity will already have an available analogue of working software including an analogue of the graph mode.

2.3. The software proposed releases the IDD user from the necessity to know in detail the IDD theory presented in the thesis. Any possible statements that the IDD method is something too mathematical, complicated and incomprehensible (and therefore the notion of loading cycle should stay as the basic one) will be groundless. Indeed, similarly to the popular program packages for finite elements modeling, the software proposed is just a 'black box' with defined objectives and with a clearly defined manner of functioning (this is additionally explained to the next practical contribution 1.2). And what exactly occurs in the 'black box' and what numerical solutions (unimaginable three or for decades ago) are done by means of the computer is another question that does not directly concern the user.

PRACTICAL CONTRIBUTIONS

1. *A practical manual* (*Section 4.1.1, subchapters 4.2 and 4.3*) *has been written which allows comparatively easy mastering and using the Ellipse software in a mass scale.*

1.1. The manual contains instructions and demos for the IDD application by means of the *Ellipse* software. It is a necessary short textbook and after studying it the IDD application can be done in a mass scale, by anyone and under any input oscillograms. By comparison, the use of many of the existing CCA criteria is understandable to their authors only, is different under different specific input oscillograms and leads to writing the next scientific paper instead to the next routine concrete application.

1.2. The manual clearly reveals the purposes and the way of function of the *Ellipse* software, as follows (in outline). Two or up to nine input R_r prototypes are entered such that the software reproduces, in smooth mode, corresponding given *S-N* lines under cyclic *r*-loadings with different values of *k*. In breaking mode (impulse mode) the given *S-N* lines directly take the part of the input R_r prototypes. The parameters f_c, f_τ, N_c and N_τ are also given. All the data to here are called Leading Data and are grouped into a so-called L-file. And the oscillograms (Current Data) are in a so-called C-file. After proceeding on these two files, the fatigue life result is obtained.

1.3. The manual presents the *Ellipse* software as consisting of two basic programs: *EllipseT* and *EllipseS*. It is explained which program under which input oscillograms is more convenient to be used. *EllipseT* can enable *T*rigonometric interpolation and is envisaged mainly for cyclic oscillograms. Besides, by request of the user, it can generate automatically sinusoidal oscillograms according to given out-of-phase angles and other data. *EllipseS* can enable *Spline* interpolation and is mainly envisaged for long, non-cyclic or random oscillograms. *EllipseT* can also be used in certain cases of such oscillograms and, if necessary, is preceded by an auxiliary program called *nv10*. The *Ellipse* software contains also an additional *EllipseC* program which substitutes *EllipseS* in case the IDD users does not prefer to have the Δs_{xy} elements but the separate points displayed on the computer screen composing a *Cloud*.

1.4. The programs are offered with a dialogue between the user and the computer in both Bulgarian and English.

2. *A separate practical computer program called Integral* (*Section 3.1.3*) *has been created to serve under r-loading only* (*under a single oscillogram*). *It is essentially simpler and more preferable than the Ellipse software.*

2.1. Every researcher who does comparative investigations of the different fatigue life assessment methods under non-cyclic *r*-loading, in general random *r*-loading, can also include the IDD method in the comparisons what is possible thanks to the simple *Integral* program.

2.2. Especially, those colleagues will be interested in the *Integral* program who are engaged with the many studies reported on regular conferences under the heading Variable Amplitude Loading (VAL). The interest will be evoked by the fact that the *Integral* program is applicable under ‘variable-amplitude loading’ without dealing with any amplitude.

3. *By means of the Integral program, IDD has been applied for fatigue life assessment under random r-loadings* (*one-component loadings*) *done by Polish researchers. The results have proved to be the best in comparison with other, CCA methods.*

3.1. One of the most competent specialists in the other methods, Dr. Jan Papuga (see http://www.pragtic.com/), has been attracted to collaborate. It has turned out that the other, CCA methods can give fatigue life results close to the IDD ones only in case a CCA criterion happens in transforming $s_{a,i}$ amplitudes depending on $s_{m,i}$, whereas such a dependence is automatically caught by IDD without any need of looking for amplitudes.

3.2. Thus, the following theoretical expectation has been confirmed: the $R_r(s)$ damage intensity, respectively the $D_r(s)$ damage function, derived by the *S-N* line i.e. derived from the corresponding cyclic *r*-loadings with the same *k*, can serve directly, without any need of rain-flow searching for cycles, for fatigue life assessment under random *r*-loading with the same *k*.

4. *An initial version of the empirical data bank of the IDD parameters* f_c, f_τ, N_c *and* N_τ *has been obtained for the first practical category of non-proportional loadings. The bank has resulted from an initial adaptation and following six verifications* (*in Chapter 5*) *based on experimental data obtained by researchers in Germany, Italy, Russia and Bulgaria* (*also in Czech Republic: in continuing verifications out of the thesis*). *By means of the EllipseT and EllipseS programs, 49 fatigue lives have been computed under quite various non-proportional loadings. There has been achieved* $(N_{cmp}/N_{exp})_{average} = 1{,}02$ *always with* $f_c = 2$ *and* $f_\tau = 3$ *except for the fifth verification where, for a steel with significantly lower static strength,* f_c *and* f_τ *have proved equal to 1. The theoretical expectation has been confirmed that the parameters* f_c *and* f_τ *will vary within a comparatively narrow interval* (1 – 3) *according to certain regularity what makes them convenient and reliable to be selected. As well, certain regularity has been outlined for the selection of* N_c *and* N_τ *comparatively to* N_r.

4.1. The data bank presented is initial and uncompleted but is a good basis for next IDD fatigue life evaluations. The inaccuracy expected would be less than 2 as an error factor ($0{,}5 < N_{cmp}/N_{exp} < 2$). This inaccuracy can always be directed to the safety side by setting proper higher values of the four IDD parameters.

4.2. At this stage, the parameters f_c and f_τ have proved to be only dependent on the material's static strength regardless of essential differences among the loadings and the specimens: for steel, f_c and f_τ are 2 and 3 in case R_m is over 700 MPa and they are 1 and 1 in case R_m is down to 410 MPa; the cast iron material in Verification (1) has shown $f_c = 2$ and $f_\tau = 3$. In next IDD applications, interpolation is possible according to these data.

4.3. It has been confirmed that the lines L_c and L_τ should be more inward to the coordinate origin than the line L_r (or L_l). This can be ensured even by the equations $N_c = N_\tau = N_r$ if, however, $f_c > 1$ and $f_\tau > 1$. Otherwise, in case f_c and f_τ approach 1, it is to set $N_c > N_r$ and $N_\tau > N_r$. Generally and approximately speaking, N_c and N_τ can be recommended so far to be in the order of $10N_r$ with a possible error in favor of safety.

5. *It has been revealed (in Chapter 6) that the fatigue loadings in machines and technical equipment in the Forest Industry have such complicatedness, poly-variety and complexity that they disclose one of the widest fields for application and manifestation of IDD. This is conditioned by the specificity of cutting and processing the wood as a material: conditions occur for all sorts of fatigue loadings which are not met at many other kinds of machines and technical equipment. Respectively, conditions occur which make the greatest difficulties trying to use CCA methods for fatigue life assessment. Thus, IDD comes at the right time and to the right place since its applicability covers namely the most complicated loadings.*

5.1. It has been revealed that, in fatigue calculation of a saw shaft, oscillograms $\sigma(t)$ and $\tau(t)$ appear which are too complicated, very specific (not met in other machine shafts), non-cyclic, random and non-proportional. In fact, every author of another method for fatigue life assessment will probably decline any invitation to apply his method after seeing the oscillograms in Fig. 6.2.3-2 in the thesis. Whereas, the IDD method is applicable namely under such conditions and therefore it will be necessitated. The knowledge from the whole thesis and mainly from the verifications in Chapter 5 will be applicable.

5.2. A calculation scheme has been applied according to which the $\sigma(t)$ oscillogram of a band-saw blade will have too complicated and random variations. They will be additionally complicated due to involvement of vibrations and other dynamic phenomena. It will be an essential scientific achievement when finally a real $\sigma(t)$ oscillogram of a band-saw blade is shown. Such an oscillogram will provide a very good example for IDD application following the verification in Chapter 3. The loading case may become classical in the books presenting the metal fatigue.